Lecture 4

Predicates, Quantifiers, Well-formed Formulas

Quantifiers

Quantifiers

Quantification expresses the extent to which a predicate is true over a range of elements.

Quantifiers

Quantification expresses the extent to which a predicate is true over a range of elements.

Universal Quantification:

Quantifiers

Quantification expresses the extent to which a predicate is true over a range of elements.

Universal Quantification:

$\forall x P(x)$ conveys that $P(x)$ is true for all values of x from a certain domain.

Quantifiers

Quantification expresses the extent to which a predicate is true over a range of elements.

Universal Quantification:

$\forall x P(x)$ conveys that $P(x)$ is true for all values of x from a certain domain.

Existential Quantification:

Quantifiers

Quantification expresses the extent to which a predicate is true over a range of elements.

Universal Quantification:

$\forall x P(x)$ conveys that $P(x)$ is true for all values of x from a certain domain.

Existential Quantification:

$\exists x P(x)$ conveys that $P(x)$ is true for some values of x from a certain domain.

Quantifiers

Quantification expresses the extent to which a predicate is true over a range of elements.

Universal Quantification:

$\forall x P(x)$ conveys that $P(x)$ is true for all values of x from a certain domain.

Existential Quantification:

$\exists x P(x)$ conveys that $P(x)$ is true for some values of x from a certain domain.

Domain is important.

Universal Quantifier

Universal Quantifier

Definition: The universal quantification of $P(x)$ is the statement

Universal Quantifier

Definition: The universal quantification of $P(x)$ is the statement

"$P(x)$ for all values of x in the domain"

Universal Quantifier

Definition: The universal quantification of $P(x)$ is the statement

$$\forall x P(x) = \text{“}P(x) \text{ for all values of } x \text{ in the domain”}$$

Universal Quantifier

Definition: The universal quantification of $P(x)$ is the statement

$$\forall x P(x) = \text{ “}P(x) \text{ for all values of } x \text{ in the domain”}$$

$\forall$ is called the universal quantifier and $\forall x P(x)$ is read as “for every x $P(x)$”.

Universal Quantifier

Definition: The universal quantification of $P(x)$ is the statement

$$\forall x P(x) = \text{"}P(x) \text{ for all values of } x \text{ in the domain"}$$

$\forall$ is called the universal quantifier and $\forall x P(x)$ is read as "for every x $P(x)$".

An element for which $\forall x P(x)$ is false is called a counterexample of $\forall x P(x)$

Universal Quantifier

Definition: The universal quantification of $P(x)$ is the statement

$$\forall x P(x) = \text{“}P(x) \text{ for all values of } x \text{ in the domain”}$$

$\forall$ is called the universal quantifier and $\forall x P(x)$ is read as “for every x $P(x)$”.

An element for which $\forall x P(x)$ is false is called a counterexample of $\forall x P(x)$

Examples:

Universal Quantifier

Definition: The universal quantification of $P(x)$ is the statement

$$\forall x P(x) = \text{ “}P(x)\text{ for all values of } x \text{ in the domain”}$$

$\forall$ is called the universal quantifier and $\forall x P(x)$ is read as “for every x $P(x)$”.

An element for which $\forall x P(x)$ is false is called a counterexample of $\forall x P(x)$

Examples:

$$P(x) = x + 1 > x$$

Universal Quantifier

Definition: The universal quantification of $P(x)$ is the statement

$$\forall x P(x) = \text{ “}P(x)\text{ for all values of } x \text{ in the domain”}$$

$\forall$ is called the universal quantifier and $\forall x P(x)$ is read as “for every x $P(x)$”.

An element for which $\forall x P(x)$ is false is called a counterexample of $\forall x P(x)$

Examples:

$P(x) = x + 1 > x$

$\forall x P(x)$ is where the domain consist of all real numbers.

Universal Quantifier

Definition: The universal quantification of $P(x)$ is the statement

$$\forall x P(x) = \text{“}P(x) \text{ for all values of } x \text{ in the domain”}$$

$\forall$ is called the universal quantifier and $\forall x P(x)$ is read as “for every x $P(x)$”.

An element for which $\forall x P(x)$ is false is called a counterexample of $\forall x P(x)$

Examples:

$P(x) = x + 1 > x$

$\forall x P(x)$ is true where the domain consist of all real numbers.

Universal Quantifier

Definition: The universal quantification of $P(x)$ is the statement

$$\forall x P(x) = \text{“}P(x) \text{ for all values of } x \text{ in the domain”}$$

$\forall$ is called the universal quantifier and $\forall x P(x)$ is read as “for every x $P(x)$”.

An element for which $\forall x P(x)$ is false is called a counterexample of $\forall x P(x)$

Examples:

$P(x) = x + 1 > x$

$\forall x P(x)$ is true where the domain consist of all real numbers.

$P(x) = x^3 \geq x$

Universal Quantifier

Definition: The universal quantification of $P(x)$ is the statement

$$\forall x P(x) = \text{ “}P(x) \text{ for all values of } x \text{ in the domain”}$$

$\forall$ is called the universal quantifier and $\forall x P(x)$ is read as “for every x $P(x)$”.

An element for which $\forall x P(x)$ is false is called a counterexample of $\forall x P(x)$

Examples:

$P(x) = x + 1 > x$

$\forall x P(x)$ is true where the domain consist of all real numbers.

$P(x) = x^3 \geq x$

$\forall x P(x)$ is where the domain consist of all positive integers.

Universal Quantifier

Definition: The universal quantification of $P(x)$ is the statement

$$\forall x P(x) = \text{"}P(x) \text{ for all values of } x \text{ in the domain"}$$

$\forall$ is called the universal quantifier and $\forall x P(x)$ is read as "for every x $P(x)$".

An element for which $\forall x P(x)$ is false is called a counterexample of $\forall x P(x)$

Examples:

$P(x) = x + 1 > x$

$\forall x P(x)$ is true where the domain consist of all real numbers.

$P(x) = x^3 \geq x$

$\forall x P(x)$ is true where the domain consist of all positive integers.

Universal Quantifier

Definition: The universal quantification of $P(x)$ is the statement

$$\forall x P(x) = \text{ "}P(x) \text{ for all values of } x \text{ in the domain"}$$

$\forall$ is called the universal quantifier and $\forall x P(x)$ is read as "for every x $P(x)$".

An element for which $\forall x P(x)$ is false is called a counterexample of $\forall x P(x)$

Examples:

$P(x) = x + 1 > x$

$\forall x P(x)$ is true where the domain consist of all real numbers.

$P(x) = x^3 \geq x$

$\forall x P(x)$ is true where the domain consist of all positive integers.

$\forall x P(x)$ is where the domain consist of all integers.

Universal Quantifier

Definition: The universal quantification of $P(x)$ is the statement

$$\forall x P(x) = \text{ “}P(x)\text{ for all values of } x \text{ in the domain”}$$

$\forall$ is called the universal quantifier and $\forall x P(x)$ is read as “for every x $P(x)$”.

An element for which $\forall x P(x)$ is false is called a counterexample of $\forall x P(x)$

Examples:

$P(x) = x + 1 > x$

$\forall x P(x)$ is true where the domain consist of all real numbers.

$P(x) = x^3 \geq x$

$\forall x P(x)$ is true where the domain consist of all positive integers.

$\forall x P(x)$ is false where the domain consist of all integers.

Universal Quantifier

Definition: The universal quantification of $P(x)$ is the statement

$$\forall x P(x) = \text{“}P(x) \text{ for all values of } x \text{ in the domain”}$$

$\forall$ is called the universal quantifier and $\forall x P(x)$ is read as “for every x $P(x)$”.

An element for which $\forall x P(x)$ is false is called a counterexample of $\forall x P(x)$

Examples:

$P(x) = x + 1 > x$

$\forall x P(x)$ is true where the domain consist of all real numbers.

$P(x) = x^3 \geq x$

$\forall x P(x)$ is true where the domain consist of all positive integers.

$\forall x P(x)$ is **false** where the domain consist of all integers. (-1 *is a counterexample.*)

Existential Quantifier

Existential Quantifier

Definition: The existential quantification of $P(x)$ is the statement

Existential Quantifier

Definition: The existential quantification of $P(x)$ is the statement

"There exists an element x in the domain such that $P(x)$."

Existential Quantifier

Definition: The existential quantification of $P(x)$ is the statement

$$\exists x P(x) = \text{"There exists an element } x \text{ in the domain such that } P(x)."$$

Existential Quantifier

Definition: The existential quantification of $P(x)$ is the statement

$$\exists x P(x) = \text{“There exists an element } x \text{ in the domain such that } P(x).\text{”}$$

$\exists$ is called the existential quantifier and $\exists x P(x)$ is read as “there exists an x such that $P(x)$”.

Existential Quantifier

Definition: The existential quantification of $P(x)$ is the statement

$$\exists x P(x) = \text{ “There exists an element } x \text{ in the domain such that } P(x).\text{”}$$

$\exists$ is called the existential quantifier and $\exists x P(x)$ is read as “there exists an x such that $P(x)$”.

Examples:

Existential Quantifier

Definition: The existential quantification of $P(x)$ is the statement

$$\exists x P(x) = \text{"There exists an element } x \text{ in the domain such that } P(x)."$$

$\exists$ is called the existential quantifier and $\exists x P(x)$ is read as "there exists an x such that $P(x)$".

Examples:

$P(x) = x$ is a prime number greater than 10^7.

Existential Quantifier

Definition: The existential quantification of $P(x)$ is the statement

$$\exists x P(x) = \text{"There exists an element } x \text{ in the domain such that } P(x)."$$

$\exists$ is called the existential quantifier and $\exists x P(x)$ is read as "there exists an x such that $P(x)$".

Examples:

$P(x) = x$ is a prime number greater than 10^7.

$\exists x P(x)$ is where the domain consist of all integers.

Existential Quantifier

Definition: The existential quantification of $P(x)$ is the statement

$$\exists x P(x) = \text{“There exists an element } x \text{ in the domain such that } P(x).\text{”}$$

$\exists$ is called the existential quantifier and $\exists x P(x)$ is read as “there exists an x such that $P(x)$”.

Examples:

$P(x) = x$ is a prime number greater than 10^7.

$\exists x P(x)$ is true where the domain consist of all integers.

Existential Quantifier

Definition: The existential quantification of $P(x)$ is the statement

$$\exists x P(x) = \text{“There exists an element } x \text{ in the domain such that } P(x).\text{”}$$

$\exists$ is called the existential quantifier and $\exists x P(x)$ is read as “there exists an x such that $P(x)$”.

Examples:

$P(x) = x$ is a prime number greater than 10^7.

$\exists x P(x)$ is true where the domain consist of all integers.

$P(x) = x = x + 1$

Existential Quantifier

Definition: The existential quantification of $P(x)$ is the statement

$$\exists x P(x) = \text{“There exists an element } x \text{ in the domain such that } P(x).\text{”}$$

$\exists$ is called the existential quantifier and $\exists x P(x)$ is read as “there exists an x such that $P(x)$”.

Examples:

$P(x) = x$ is a prime number greater than 10^7.

$\exists x P(x)$ is true where the domain consist of all integers.

$P(x) = x = x + 1$

$\exists x P(x)$ is where the domain consist of all real numbers.

Existential Quantifier

Definition: The existential quantification of $P(x)$ is the statement

$$\exists x P(x) = \text{ "There exists an element } x \text{ in the domain such that } P(x)\text{."}$$

$\exists$ is called the existential quantifier and $\exists x P(x)$ is read as "there exists an x such that $P(x)$".

Examples:

$P(x) = x$ is a prime number greater than 10^7.

$\exists x P(x)$ is true where the domain consist of all integers.

$$P(x) = x = x + 1$$

$\exists x P(x)$ is false where the domain consist of all real numbers.

Truth Values of Quantifiers

Truth Values of Quantifiers

Let $P(x)$ be a propositional function and domain $= \{x_1, x_2, \ldots, x_n, \ldots\}$:

Truth Values of Quantifiers

Let $P(x)$ be a propositional function and domain $= \{x_1, x_2, \ldots, x_n, \ldots\}$:

- $\forall x P(x)$ is true if $P(x_1) \wedge P(x_2) \wedge \ldots \wedge P(x_n) \wedge \ldots$ is true, is **false** otherwise.

Truth Values of Quantifiers

Let $P(x)$ be a propositional function and domain $= \{x_1, x_2, \ldots, x_n, \ldots\}$:

- $\forall x P(x)$ is true if $P(x_1) \wedge P(x_2) \wedge \ldots \wedge P(x_n) \wedge \ldots$ is true, is **false** otherwise.
- $\exists x P(x)$ is true if $P(x_1) \vee P(x_2) \vee \ldots \vee P(x_n) \vee \ldots$ is true, is **false** otherwise.

Well-formed Formulas

Well-formed Formulas

Definition*:

Well-formed Formulas

Definition*: 1. Propositional variables such as $p, q, r,$ etc., are WFFs.

Well-formed Formulas

Definition*: 1. Propositional variables such as $p, q, r,$ etc., are WFFs.

2. Propositional functions such as $P(x), Q(x, y), R(x, y, z),$ etc., are WFF.

Well-formed Formulas

Definition*: 1. Propositional variables such as $p, q, r,$ etc., are WFFs.

2. Propositional functions such as $P(x), Q(x, y), R(x, y, z),$ etc., are WFF.

3. If α and β are two WFFs, then $(\neg\alpha)$, $(\alpha \vee \beta)$, $(\alpha \wedge \beta)$, $(\alpha \rightarrow \beta)$, and $(\alpha \leftrightarrow \beta)$

Well-formed Formulas

Definition*: 1. Propositional variables such as $p, q, r,$ etc., are WFFs.

2. Propositional functions such as $P(x), Q(x, y), R(x, y, z),$ etc., are WFF.

3. If α and β are two WFFs, then $(\neg\alpha)$, $(\alpha \vee \beta)$, $(\alpha \wedge \beta)$, $(\alpha \rightarrow \beta)$, and $(\alpha \leftrightarrow \beta)$ are WFFs.

Well-formed Formulas

Definition*: 1. Propositional variables such as $p, q, r,$ etc., are WFFs.

2. Propositional functions such as $P(x), Q(x, y), R(x, y, z),$ etc., are WFF.

3. If α and β are two WFFs, then $(\neg\alpha)$, $(\alpha \vee \beta)$, $(\alpha \wedge \beta)$, $(\alpha \rightarrow \beta)$, and $(\alpha \leftrightarrow \beta)$ are WFFs.

4. If α is a WFF where all occurrences of x are free

Well-formed Formulas

Definition*:

1. Propositional variables such as $p, q, r,$ etc., are WFFs.
2. Propositional functions such as $P(x), Q(x, y), R(x, y, z),$ etc., are WFF.
3. If α and β are two WFFs, then $(\neg\alpha)$, $(\alpha \vee \beta)$, $(\alpha \wedge \beta)$, $(\alpha \rightarrow \beta)$, and $(\alpha \leftrightarrow \beta)$ are WFFs.
4. If α is a WFF where all occurrences of x are free, then $\forall x\alpha$ and $\exists x\alpha$ are WFFs.

Well-formed Formulas

Definition*: 1. Propositional variables such as $p, q, r,$ etc., are WFFs.

2. Propositional functions such as $P(x), Q(x, y), R(x, y, z),$ etc., are WFF.

3. If α and β are two WFFs, then $(\neg\alpha)$, $(\alpha \vee \beta)$, $(\alpha \wedge \beta)$, $(\alpha \rightarrow \beta)$, and $(\alpha \leftrightarrow \beta)$ are WFFs.

4. If α is a WFF where all occurrences of x are free, then $\forall x\alpha$ and $\exists x\alpha$ are WFFs.

Example:

Well-formed Formulas

Definition*: 1. Propositional variables such as $p, q, r,$ etc., are WFFs.

2. Propositional functions such as $P(x), Q(x, y), R(x, y, z),$ etc., are WFF.

3. If α and β are two WFFs, then $(\neg\alpha)$, $(\alpha \vee \beta)$, $(\alpha \wedge \beta)$, $(\alpha \rightarrow \beta)$, and $(\alpha \leftrightarrow \beta)$ are WFFs.

4. If α is a WFF where all occurrences of x are free, then $\forall x\alpha$ and $\exists x\alpha$ are WFFs.

Example: $\forall x(p \vee (\neg Q(x)))$ is a WFF.

Well-formed Formulas

Definition*: 1. Propositional variables such as $p, q, r,$ etc., are WFFs.

2. Propositional functions such as $P(x), Q(x, y), R(x, y, z),$ etc., are WFF.

3. If α and β are two WFFs, then $(\neg\alpha)$, $(\alpha \vee \beta)$, $(\alpha \wedge \beta)$, $(\alpha \rightarrow \beta)$, and $(\alpha \leftrightarrow \beta)$ are WFFs.

4. If α is a WFF where all occurrences of x are free, then $\forall x\alpha$ and $\exists x\alpha$ are WFFs.

Example: $\forall x(p \vee (\neg Q(x)))$ is a WFF.

p

Well-formed Formulas

Definition*: 1. Propositional variables such as $p, q, r,$ etc., are WFFs.

2. Propositional functions such as $P(x), Q(x, y), R(x, y, z),$ etc., are WFF.

3. If α and β are two WFFs, then $(\neg\alpha)$, $(\alpha \vee \beta)$, $(\alpha \wedge \beta)$, $(\alpha \rightarrow \beta)$, and $(\alpha \leftrightarrow \beta)$ are WFFs.

4. If α is a WFF where all occurrences of x are free, then $\forall x\alpha$ and $\exists x\alpha$ are WFFs.

Example: $\forall x(p \vee (\neg Q(x)))$ is a WFF.

$p \quad Q(x)$

Well-formed Formulas

Definition*:

1. Propositional variables such as $p, q, r,$ etc., are WFFs.
2. Propositional functions such as $P(x), Q(x, y), R(x, y, z),$ etc., are WFF.
3. If α and β are two WFFs, then $(\neg\alpha)$, $(\alpha \vee \beta)$, $(\alpha \wedge \beta)$, $(\alpha \rightarrow \beta)$, and $(\alpha \leftrightarrow \beta)$ are WFFs.
4. If α is a WFF where all occurrences of x are free, then $\forall x\alpha$ and $\exists x\alpha$ are WFFs.

Example: $\forall x(p \vee (\neg Q(x)))$ is a WFF.

$$p \qquad Q(x) \qquad (\neg Q(x))$$

Well-formed Formulas

Definition*: 1. Propositional variables such as $p, q, r,$ etc., are WFFs.

2. Propositional functions such as $P(x), Q(x, y), R(x, y, z),$ etc., are WFF.

3. If α and β are two WFFs, then $(\neg\alpha)$, $(\alpha \vee \beta)$, $(\alpha \wedge \beta)$, $(\alpha \to \beta)$, and $(\alpha \leftrightarrow \beta)$ are WFFs.

4. If α is a WFF where all occurrences of x are free, then $\forall x\alpha$ and $\exists x\alpha$ are WFFs.

Example: $\forall x(p \vee (\neg Q(x)))$ is a WFF.

$$\begin{tikzcd}
p \arrow[rrr, bend left] & Q(x) \arrow[r, bend left] & (\neg Q(x)) \arrow[r, bend left] & (p \vee (\neg Q(x)))
\end{tikzcd}$$

Well-formed Formulas

Definition*: 1. Propositional variables such as $p, q, r,$ etc., are WFFs.

2. Propositional functions such as $P(x), Q(x, y), R(x, y, z),$ etc., are WFF.

3. If α and β are two WFFs, then $(\neg\alpha)$, $(\alpha \vee \beta)$, $(\alpha \wedge \beta)$, $(\alpha \rightarrow \beta)$, and $(\alpha \leftrightarrow \beta)$ are WFFs.

4. If α is a WFF where all occurrences of x are free, then $\forall x\alpha$ and $\exists x\alpha$ are WFFs.

Example: $\forall x(p \vee (\neg Q(x)))$ is a WFF.

$$\begin{tikzcd}
p \arrow[rrr, bend left] & Q(x) \arrow[r, bend left] & (\neg Q(x)) \arrow[r, bend left] & (p \vee (\neg Q(x))) \arrow[r, bend left] & \forall x(p \vee (\neg Q(x)))
\end{tikzcd}$$

Well-formed Formulas

Definition*: 1. Propositional variables such as $p, q, r,$ etc., are WFFs.

2. Propositional functions such as $P(x), Q(x, y), R(x, y, z),$ etc., are WFF.

3. If α and β are two WFFs, then $(\neg\alpha)$, $(\alpha \vee \beta)$, $(\alpha \wedge \beta)$, $(\alpha \to \beta)$, and $(\alpha \leftrightarrow \beta)$ are WFFs.

4. If α is a WFF where all occurrences of x are free, then $\forall x\alpha$ and $\exists x\alpha$ are WFFs.

Example: $\forall x(p \vee (\neg Q(x)))$ is a WFF.

$$\begin{tikzcd}
p \arrow[rrr, bend left] & Q(x) \arrow[r, bend left] & (\neg Q(x)) \arrow[r, bend left] & (p \vee (\neg Q(x)) \arrow[r, bend left] & \forall x(p \vee (\neg Q(x))
\end{tikzcd}$$

$\forall x(\exists x(P(x)))$ is not a WFF.

Well-formed Formulas

Definition*: 1. Propositional variables such as $p, q, r,$ etc., are WFFs.

2. Propositional functions such as $P(x), Q(x, y), R(x, y, z),$ etc., are WFF.

3. If α and β are two WFFs, then $(\neg\alpha)$, $(\alpha \lor \beta)$, $(\alpha \land \beta)$, $(\alpha \to \beta)$, and $(\alpha \leftrightarrow \beta)$ are WFFs.

4. If α is a WFF where all occurrences of x are free, then $\forall x \alpha$ and $\exists x \alpha$ are WFFs.

Example: $\forall x(p \lor (\neg Q(x)))$ is a WFF.

$$\begin{tikzcd}
p \arrow[rrr, bend left] & Q(x) \arrow[r, bend left] & (\neg Q(x)) \arrow[r, bend left] & (p \lor (\neg Q(x)) \arrow[r, bend left] & \forall x(p \lor (\neg Q(x))
\end{tikzcd}$$

$\forall x(\exists x(P(x)))$ is not a WFF.

*WFF in FOL are slightly different from how they are defined here.

Precedence of Quantifier

Precedence of Quantifier

$\forall$ and $\exists$ have higher precedence than all the logical operators.

Precedence of Quantifier

$\forall$ and $\exists$ have higher precedence than all the logical operators.

$\forall x P(x) \vee Q(x)$ is $(\forall x P(x)) \vee Q(x)$ not $\forall x (P(x) \vee Q(x))$

Precedence of Quantifier

$\forall$ and $\exists$ have higher precedence than all the logical operators.

$\forall x P(x) \vee Q(x)$ is $(\forall x P(x)) \vee Q(x)$ not $\forall x(P(x) \vee Q(x))$

$\forall x P(x) \rightarrow \exists y Q(y)$ is $(\forall x P(x)) \rightarrow (\exists y Q(y))$ not $\forall x(P(x) \rightarrow \exists y Q(y))$

Binding Variables

Binding Variables

Example:

Binding Variables

Example:

$$(x + y) = 1$$

Binding Variables

Example:

$$\underbrace{(x + y) = 1}$$

Not a proposition, both x & y are free.

Binding Variables

- An occurrence of a variable that is not bound by a quantifier is called free.

Example:

$$\underbrace{(x + y) = 1}$$

Not a proposition, both x & y are free.

Binding Variables

- An occurrence of a variable that is not bound by a quantifier is called free.

Example:

$$\exists x \, \underbrace{(x + y) = 1}$$

Not a proposition, both x & y are free.

Binding Variables

- An occurrence of a variable that is not bound by a quantifier is called free.

Example:

$$\exists x\,(x + y) = 1$$

Not a proposition, both x & y are free.

Not a proposition, x is bound but y is free.

Binding Variables

- An occurrence of a variable that is not bound by a quantifier is called free.
- An occurrence of a variable that is bound by a quantifier is called bound.

Example:

$$\exists x\,(x + y) = 1$$

Not a proposition, both x & y are free.

Not a proposition, x is bound but y is free.

Binding Variables

- An occurrence of a variable that is not bound by a quantifier is called free.
- An occurrence of a variable that is bound by a quantifier is called bound.

Example:

$$\forall y \, \exists x \, (x + y) = 1$$

Not a proposition, both x & y are free.

Not a proposition, x is bound but y is free.

Binding Variables

- An occurrence of a variable that is not bound by a quantifier is called free.
- An occurrence of a variable that is bound by a quantifier is called bound.

Example:

$$\forall y \, \exists x \, (x + y) = 1$$

Not a proposition, both x & y are free.

Not a proposition, x is bound but y is free.

A proposition as no variable is free.

Scope

Scope

Example:

Scope

Example:

$$\exists x\,(P(x) \land Q(x)) \lor \forall x\, R(x)$$

Scope

Example:

$$\exists x\ (\underline{P(x) \wedge Q(x)}) \vee \forall x\ R(x)$$

Scope of $\exists x$ is $P(x) \wedge Q(x)$

Scope

Example:

$$\exists x\,(\underline{P(x) \wedge Q(x)}) \vee \forall x\,\underline{R(x)}$$

Scope of $\exists x$ is $P(x) \wedge Q(x)$

Scope of $\forall x$ is $R(x)$

Scope

The part of a logical expression where a quantifier is applied is called the scope of the quantifier.

Example:

$$\exists x\ (\underline{P(x) \wedge Q(x)}) \vee \forall x\ \underline{R(x)}$$

Scope of $\exists x$ is $P(x) \wedge Q(x)$

Scope of $\forall x$ is $R(x)$

Scope

The part of a logical expression where a quantifier is applied is called the scope of the quantifier.

Example:

$$\exists x\ (\underline{P(x) \wedge Q(x)}) \vee \forall x\ \underline{R(x)}$$

Scope of $\exists x$ is $P(x) \wedge Q(x)$

Scope of $\forall x$ is $R(x)$

Tip: To avoid confusion, use different variables:

Scope

The part of a logical expression where a quantifier is applied is called the scope of the quantifier.

Example:

$$\exists x\, \underline{(P(x) \wedge Q(x))} \vee \forall x\, \underline{R(x)}$$

Scope of $\exists x$ is $P(x) \wedge Q(x)$

Scope of $\forall x$ is $R(x)$

Tip: To avoid confusion, use different variables:

$$\exists x\, (P(x) \wedge Q(x)) \vee \forall y\, R(y)$$

Logical Equivalence of WFFs

Logical Equivalence of WFFs

Definition:

Logical Equivalence of WFFs

Definition: Two WFFs containing same set of propositional variables and propositional

Logical Equivalence of WFFs

Definition: Two WFFs containing same set of propositional variables and propositional functions

Logical Equivalence of WFFs

Definition: Two WFFs containing same set of propositional variables and propositional functions with no free variables are logically equivalent if and only if

Logical Equivalence of WFFs

Definition: Two WFFs containing same set of propositional variables and propositional functions with no free variables are logically equivalent if and only if they have the same

Logical Equivalence of WFFs

Definition: Two WFFs containing same set of propositional variables and propositional functions with no free variables are logically equivalent if and only if they have the same truth value,

Logical Equivalence of WFFs

Definition: Two WFFs containing same set of propositional variables and propositional functions with no free variables are logically equivalent if and only if they have the same truth value, no matter which propositions and predicates are substituted in WFFs and

Logical Equivalence of WFFs

Definition: Two WFFs containing same set of propositional variables and propositional functions with no free variables are logically equivalent if and only if they have the same truth value, no matter which propositions and predicates are substituted in WFFs and which domain is used for variables.

Logical Equivalence of WFFs

Definition: Two WFFs containing same set of propositional variables and propositional functions with no free variables are logically equivalent if and only if they have the same truth value, no matter which propositions and predicates are substituted in WFFs and which domain is used for variables.

Example:

Logical Equivalence of WFFs

Definition: Two WFFs containing same set of propositional variables and propositional functions with no free variables are logically equivalent if and only if they have the same truth value, no matter which propositions and predicates are substituted in WFFs and which domain is used for variables.

Example: $\forall x(P(x) \land Q(x))$ is logically equivalent to $\forall xP(x) \land \forall xQ(x)$

Logical Equivalence of WFFs

Definition: Two WFFs containing same set of propositional variables and propositional functions with no free variables are logically equivalent if and only if they have the same truth value, no matter which propositions and predicates are substituted in WFFs and which domain is used for variables.

Example: $\forall x(P(x) \land Q(x))$ is logically equivalent to $\forall x P(x) \land \forall x Q(x)$

(Check proof in the book if you are interested.)

Negating Quantified Expressions

Negating Quantified Expressions

$q =$ "Every student in this class has taken ICS."

Negating Quantified Expressions

$q =$ "Every student in this class has taken ICS."

$q = \forall x P(x)$,

Negating Quantified Expressions

$q =$ "Every student in this class has taken ICS."

$q = \forall x P(x)$, where $P(x)$ = "x has taken ICS"

Negating Quantified Expressions

$q =$ "Every student in this class has taken ICS."

$q = \forall x P(x)$, where $P(x)$ = "x has taken ICS" and domain is the set of students in this class.

Negating Quantified Expressions

$q =$ "Every student in this class has taken ICS."

$q = \forall x P(x)$, where $P(x) =$ "x has taken ICS" and domain is the set of students in this class.

What's the negation of q?

Negating Quantified Expressions

$q =$ "Every student in this class has taken ICS."

$q = \forall x P(x)$, where $P(x) =$ "x has taken ICS" and domain is the set of students in this class.

What's the negation of q?

$\neg q =$ "It is not the case that every student in this class has taken ICS."

Negating Quantified Expressions

$q =$ "Every student in this class has taken ICS."

$q = \forall x P(x)$, where $P(x) =$ "x has taken ICS" and domain is the set of students in this class.

What's the negation of q?

$\neg q =$ "It is not the case that every student in this class has taken ICS."

$=$ "There is a student in this class who has not taken ICS."

Negating Quantified Expressions

$q =$ "Every student in this class has taken ICS."

$q = \forall x P(x)$, where $P(x)$ = "x has taken ICS" and domain is the set of students in this class.

What's the negation of q?

$\neg q =$ "It is not the case that every student in this class has taken ICS."

$=$ "There is a student in this class who has not taken ICS."

$= \exists x \neg P(x)$

Negating Quantified Expressions

$q =$ "Every student in this class has taken ICS."

$q = \forall x P(x)$, where $P(x) =$ "x has taken ICS" and domain is the set of students in this class.

What's the negation of q?

$\neg q =$ "It is not the case that every student in this class has taken ICS."

$=$ "There is a student in this class who has not taken ICS."

$= \exists x \neg P(x)$

This illustrates the logical equivalence:

Negating Quantified Expressions

q = "Every student in this class has taken ICS."

$q = \forall x P(x)$, where $P(x)$ = "x has taken ICS" and domain is the set of students in this class.

What's the negation of q?

$\neg q$ = "It is not the case that every student in this class has taken ICS."

= "There is a student in this class who has not taken ICS."

$= \exists x \neg P(x)$

This illustrates the logical equivalence:

$$\neg \forall x P(x) \equiv \exists x \neg P(x)$$

Negating Quantified Expressions

Negating Quantified Expressions

$q =$ "There's a student in M4C who had 100% attendance in ICS."

Negating Quantified Expressions

$q =$ "There's a student in M4C who had 100% attendance in ICS."

$q = \exists x P(x)$,

Negating Quantified Expressions

$q =$ "There's a student in M4C who had 100% attendance in ICS."

$q = \exists x P(x)$, where $P(x)$ = "x had 100% attendance in ICS" and domain is set of students in M4C.

Negating Quantified Expressions

$q =$ "There's a student in M4C who had 100% attendance in ICS."

$q = \exists x P(x)$, where $P(x)$ = "x had 100% attendance in ICS" and domain is set of students in M4C.

What's the negation of q?

Negating Quantified Expressions

$q =$ "There's a student in M4C who had 100% attendance in ICS."

$q = \exists x P(x)$, where $P(x)$ = "x had 100% attendance in ICS" and domain is set of students in M4C.

What's the negation of q?

$\neg q =$ "It is not the case that there's a student in M4C who had 100% attendance in ICS."

Negating Quantified Expressions

$q =$ "There's a student in M4C who had 100% attendance in ICS."

$q = \exists x P(x)$, where $P(x)$ = "x had 100% attendance in ICS" and domain is set of students in M4C.

What's the negation of q?

$\neg q =$ "It is not the case that there's a student in M4C who had 100% attendance in ICS."

$=$ "Every student in M4C did not have 100% attendance in ICS."

Negating Quantified Expressions

$q =$ "There's a student in M4C who had 100% attendance in ICS."

$q = \exists x P(x)$, where $P(x) =$ "x had 100% attendance in ICS" and domain is set of students in M4C.

What's the negation of q?

$\neg q =$ "It is not the case that there's a student in M4C who had 100% attendance in ICS."

$=$ "Every student in M4C did not have 100% attendance in ICS."

$= \forall x \neg P(x)$

Negating Quantified Expressions

$q =$ "There's a student in M4C who had 100% attendance in ICS."

$q = \exists x P(x)$, where $P(x)$ = "x had 100% attendance in ICS" and domain is set of students in M4C.

What's the negation of q?

$\neg q =$ "It is not the case that there's a student in M4C who had 100% attendance in ICS."

$=$ "Every student in M4C did not have 100% attendance in ICS."

$= \forall x \neg P(x)$

This illustrates the logical equivalence:

$$\neg \exists x P(x) \equiv \forall x \neg P(x)$$