

Lecture 4

Predicates, Quantifiers, Well-formed Formulas

Quantifiers

Quantifiers

Quantification expresses the extent to which a predicate is true over a range of elements.

Quantifiers

Quantification expresses the extent to which a predicate is true over a range of elements.

Universal Quantification:

Quantifiers

Quantification expresses the extent to which a predicate is true over a range of elements.

Universal Quantification:

$\forall x P(x)$ conveys that $P(x)$ is true for all values of x from a certain domain.

Quantifiers

Quantification expresses the extent to which a predicate is true over a range of elements.

Universal Quantification:

$\forall xP(x)$ conveys that $P(x)$ is true for all values of x from a certain domain.

Existential Quantification:

Quantifiers

Quantification expresses the extent to which a predicate is true over a range of elements.

Universal Quantification:

$\forall xP(x)$ conveys that $P(x)$ is true for all values of x from a certain domain.

Existential Quantification:

$\exists xP(x)$ conveys that $P(x)$ is true for some values of x from a certain domain.

Quantifiers

Quantification expresses the extent to which a predicate is true over a range of elements.

Universal Quantification:

$\forall xP(x)$ conveys that $P(x)$ is true for all values of x from a certain domain.

Existential Quantification:

$\exists xP(x)$ conveys that $P(x)$ is true for some values of x from a certain domain.

Domain is important.



Universal Quantifier

Universal Quantifier

Definition: The **universal quantification** of $P(x)$ is the statement

Universal Quantifier

Definition: The **universal quantification** of $P(x)$ is the statement
" $P(x)$ for all values of x in the domain"

Universal Quantifier

Definition: The **universal quantification** of $P(x)$ is the statement

$$\forall x P(x) = \text{"}P(x) \text{ for all values of } x \text{ in the domain"}$$

Universal Quantifier

Definition: The **universal quantification** of $P(x)$ is the statement

$$\forall x P(x) = \text{"}P(x) \text{ for all values of } x \text{ in the domain"}$$

\forall is called the universal quantifier and $\forall x P(x)$ is read as "for every x $P(x)$ ".

Universal Quantifier

Definition: The **universal quantification** of $P(x)$ is the statement

$$\forall x P(x) = "P(x) \text{ for all values of } x \text{ in the domain}"$$

\forall is called the universal quantifier and $\forall x P(x)$ is read as "for every x $P(x)$ ".

An element for which $\forall x P(x)$ is false is called a **counterexample** of $\forall x P(x)$

Universal Quantifier

Definition: The **universal quantification** of $P(x)$ is the statement

$$\forall x P(x) = \text{"}P(x) \text{ for all values of } x \text{ in the domain"}$$

\forall is called the universal quantifier and $\forall x P(x)$ is read as "for every x $P(x)$ ".

An element for which $\forall x P(x)$ is false is called a **counterexample** of $\forall x P(x)$

Examples:

Universal Quantifier

Definition: The **universal quantification** of $P(x)$ is the statement

$$\forall x P(x) = \text{"}P(x) \text{ for all values of } x \text{ in the domain"}$$

\forall is called the universal quantifier and $\forall x P(x)$ is read as "for every x $P(x)$ ".

An element for which $\forall x P(x)$ is false is called a **counterexample** of $\forall x P(x)$

Examples:

$$P(x) = x + 1 > x$$

Universal Quantifier

Definition: The **universal quantification** of $P(x)$ is the statement

$$\forall x P(x) = \text{"}P(x) \text{ for all values of } x \text{ in the domain"}$$

\forall is called the universal quantifier and $\forall x P(x)$ is read as "for every x $P(x)$ ".

An element for which $\forall x P(x)$ is false is called a **counterexample** of $\forall x P(x)$

Examples:

$$P(x) = x + 1 > x$$

$\forall x P(x)$ is where the domain consist of all real numbers.

Universal Quantifier

Definition: The **universal quantification** of $P(x)$ is the statement

$$\forall x P(x) = "P(x) \text{ for all values of } x \text{ in the domain}"$$

\forall is called the universal quantifier and $\forall x P(x)$ is read as "for every x $P(x)$ ".

An element for which $\forall x P(x)$ is false is called a **counterexample** of $\forall x P(x)$

Examples:

$$P(x) = x + 1 > x$$

$\forall x P(x)$ is **true** where the domain consist of all real numbers.

Universal Quantifier

Definition: The **universal quantification** of $P(x)$ is the statement

$$\forall x P(x) = \text{"}P(x) \text{ for all values of } x \text{ in the domain"}$$

\forall is called the universal quantifier and $\forall x P(x)$ is read as "for every x $P(x)$ ".

An element for which $\forall x P(x)$ is false is called a **counterexample** of $\forall x P(x)$

Examples:

$$P(x) = x + 1 > x$$

$\forall x P(x)$ is **true** where the domain consist of all real numbers.

$$P(x) = x^3 \geq x$$

Universal Quantifier

Definition: The **universal quantification** of $P(x)$ is the statement

$$\forall x P(x) = \text{"}P(x) \text{ for all values of } x \text{ in the domain"}$$

\forall is called the universal quantifier and $\forall x P(x)$ is read as "for every x $P(x)$ ".

An element for which $\forall x P(x)$ is false is called a **counterexample** of $\forall x P(x)$

Examples:

$$P(x) = x + 1 > x$$

$\forall x P(x)$ is **true** where the domain consist of all real numbers.

$$P(x) = x^3 \geq x$$

$\forall x P(x)$ is where the domain consist of all positive integers.

Universal Quantifier

Definition: The **universal quantification** of $P(x)$ is the statement

$$\forall x P(x) = \text{"}P(x) \text{ for all values of } x \text{ in the domain"}$$

\forall is called the universal quantifier and $\forall x P(x)$ is read as "for every x $P(x)$ ".

An element for which $\forall x P(x)$ is false is called a **counterexample** of $\forall x P(x)$

Examples:

$$P(x) = x + 1 > x$$

$\forall x P(x)$ is **true** where the domain consist of all real numbers.

$$P(x) = x^3 \geq x$$

$\forall x P(x)$ is **true** where the domain consist of all positive integers.

Universal Quantifier

Definition: The **universal quantification** of $P(x)$ is the statement

$$\forall x P(x) = \text{"}P(x) \text{ for all values of } x \text{ in the domain"}$$

\forall is called the universal quantifier and $\forall x P(x)$ is read as "for every x $P(x)$ ".

An element for which $\forall x P(x)$ is false is called a **counterexample** of $\forall x P(x)$

Examples:

$$P(x) = x + 1 > x$$

$\forall x P(x)$ is **true** where the domain consist of all real numbers.

$$P(x) = x^3 \geq x$$

$\forall x P(x)$ is **true** where the domain consist of all positive integers.

$\forall x P(x)$ is where the domain consist of all integers.

Universal Quantifier

Definition: The **universal quantification** of $P(x)$ is the statement

$$\forall x P(x) = \text{"}P(x) \text{ for all values of } x \text{ in the domain"}$$

\forall is called the universal quantifier and $\forall x P(x)$ is read as "for every x $P(x)$ ".

An element for which $\forall x P(x)$ is false is called a **counterexample** of $\forall x P(x)$

Examples:

$$P(x) = x + 1 > x$$

$\forall x P(x)$ is **true** where the domain consist of all real numbers.

$$P(x) = x^3 \geq x$$

$\forall x P(x)$ is **true** where the domain consist of all positive integers.

$\forall x P(x)$ is **false** where the domain consist of all integers.

Universal Quantifier

Definition: The **universal quantification** of $P(x)$ is the statement

$$\forall x P(x) = "P(x) \text{ for all values of } x \text{ in the domain}"$$

\forall is called the universal quantifier and $\forall x P(x)$ is read as "for every x $P(x)$ ".

An element for which $\forall x P(x)$ is false is called a **counterexample** of $\forall x P(x)$

Examples:

$$P(x) = x + 1 > x$$

$\forall x P(x)$ is **true** where the domain consist of all real numbers.

$$P(x) = x^3 \geq x$$

$\forall x P(x)$ is **true** where the domain consist of all positive integers.

$\forall x P(x)$ is **false** where the domain consist of all integers. (-1 is a counterexample.)

Existential Quantifier

Existential Quantifier

Definition: The **existential quantification** of $P(x)$ is the statement

Existential Quantifier

Definition: The **existential quantification** of $P(x)$ is the statement

“There exists an element x in the domain such that $P(x)$.”

Existential Quantifier

Definition: The **existential quantification** of $P(x)$ is the statement

$\exists xP(x) =$ "There exists an element x in the domain such that $P(x)$."

Existential Quantifier

Definition: The **existential quantification** of $P(x)$ is the statement

$$\exists xP(x) = \text{"There exists an element } x \text{ in the domain such that } P(x)."$$

\exists is called the existential quantifier and $\exists xP(x)$ is read as "there exists an x such that $P(x)$ ".

Existential Quantifier

Definition: The **existential quantification** of $P(x)$ is the statement

$$\exists xP(x) = \text{"There exists an element } x \text{ in the domain such that } P(x)."$$

\exists is called the existential quantifier and $\exists xP(x)$ is read as "there exists an x such that $P(x)$ ".

Examples:

Existential Quantifier

Definition: The **existential quantification** of $P(x)$ is the statement

$$\exists x P(x) = \text{"There exists an element } x \text{ in the domain such that } P(x)."$$

\exists is called the existential quantifier and $\exists x P(x)$ is read as "there exists an x such that $P(x)$ ".

Examples:

$$P(x) = x \text{ is a prime number greater than } 10^7.$$

Existential Quantifier

Definition: The **existential quantification** of $P(x)$ is the statement

$$\exists x P(x) = \text{"There exists an element } x \text{ in the domain such that } P(x)."$$

\exists is called the existential quantifier and $\exists x P(x)$ is read as "there exists an x such that $P(x)$ ".

Examples:

$P(x) = x$ is a prime number greater than 10^7 .

$\exists x P(x)$ is where the domain consist of all integers.

Existential Quantifier

Definition: The **existential quantification** of $P(x)$ is the statement

$$\exists x P(x) = \text{"There exists an element } x \text{ in the domain such that } P(x)."$$

\exists is called the existential quantifier and $\exists x P(x)$ is read as "there exists an x such that $P(x)$ ".

Examples:

$P(x) = x$ is a prime number greater than 10^7 .

$\exists x P(x)$ is **true** where the domain consist of all integers.

Existential Quantifier

Definition: The **existential quantification** of $P(x)$ is the statement

$$\exists x P(x) = \text{"There exists an element } x \text{ in the domain such that } P(x)."$$

\exists is called the existential quantifier and $\exists x P(x)$ is read as "there exists an x such that $P(x)$ ".

Examples:

$P(x) = x$ is a prime number greater than 10^7 .

$\exists x P(x)$ is **true** where the domain consist of all integers.

$$P(x) = x = x + 1$$

Existential Quantifier

Definition: The **existential quantification** of $P(x)$ is the statement

$$\exists x P(x) = \text{"There exists an element } x \text{ in the domain such that } P(x)."$$

\exists is called the existential quantifier and $\exists x P(x)$ is read as "there exists an x such that $P(x)$ ".

Examples:

$P(x) = x$ is a prime number greater than 10^7 .

$\exists x P(x)$ is **true** where the domain consist of all integers.

$$P(x) = x = x + 1$$

$\exists x P(x)$ is where the domain consist of all real numbers.

Existential Quantifier

Definition: The **existential quantification** of $P(x)$ is the statement

$$\exists x P(x) = \text{"There exists an element } x \text{ in the domain such that } P(x)."$$

\exists is called the existential quantifier and $\exists x P(x)$ is read as "there exists an x such that $P(x)$ ".

Examples:

$P(x) = x$ is a prime number greater than 10^7 .

$\exists x P(x)$ is **true** where the domain consist of all integers.

$$P(x) = x = x + 1$$

$\exists x P(x)$ is **false** where the domain consist of all real numbers.

Truth Values of Quantifiers

Truth Values of Quantifiers

Let $P(x)$ be a propositional function and domain $= \{x_1, x_2, \dots, x_n, \dots\}$:

Truth Values of Quantifiers

Let $P(x)$ be a propositional function and domain = $\{x_1, x_2, \dots, x_n, \dots\}$:

- $\forall x P(x)$ is **true** if $P(x_1) \wedge P(x_2) \wedge \dots \wedge P(x_n) \wedge \dots$ is true, is **false** otherwise.

Truth Values of Quantifiers

Let $P(x)$ be a propositional function and domain = $\{x_1, x_2, \dots, x_n, \dots\}$:

- $\forall xP(x)$ is **true** if $P(x_1) \wedge P(x_2) \wedge \dots \wedge P(x_n) \wedge \dots$ is true, is **false** otherwise.
- $\exists xP(x)$ is **true** if $P(x_1) \vee P(x_2) \vee \dots \vee P(x_n) \vee \dots$ is true, is **false** otherwise.

Well-formed Formulas

Well-formed Formulas

Definition*:

Well-formed Formulas

Definition*: 1. Propositional variables such as *p, q, r*, etc., are WFFs.

Well-formed Formulas

Definition*: 1. Propositional variables such as p, q, r , etc., are WFFs.
2. Propositional functions such as $P(x), Q(x, y), R(x, y, z)$, etc., are WFF.

Well-formed Formulas

- Definition*:**
1. Propositional variables such as p, q, r , etc., are WFFs.
 2. Propositional functions such as $P(x), Q(x, y), R(x, y, z)$, etc., are WFF.
 3. If α and β are two WFFs, then $(\neg\alpha), (\alpha \vee \beta), (\alpha \wedge \beta), (\alpha \rightarrow \beta)$, and $(\alpha \leftrightarrow \beta)$

Well-formed Formulas

- Definition*:**
1. Propositional variables such as p, q, r , etc., are WFFs.
 2. Propositional functions such as $P(x), Q(x, y), R(x, y, z)$, etc., are WFF.
 3. If α and β are two WFFs, then $(\neg\alpha), (\alpha \vee \beta), (\alpha \wedge \beta), (\alpha \rightarrow \beta)$, and $(\alpha \leftrightarrow \beta)$ are WFFs.

Well-formed Formulas

- Definition*:**
1. Propositional variables such as p, q, r , etc., are WFFs.
 2. Propositional functions such as $P(x), Q(x, y), R(x, y, z)$, etc., are WFF.
 3. If α and β are two WFFs, then $(\neg\alpha), (\alpha \vee \beta), (\alpha \wedge \beta), (\alpha \rightarrow \beta)$, and $(\alpha \leftrightarrow \beta)$ are WFFs.
 4. If α is a WFF where all occurrences of x are free

Well-formed Formulas

- Definition*:**
1. Propositional variables such as p, q, r , etc., are WFFs.
 2. Propositional functions such as $P(x), Q(x, y), R(x, y, z)$, etc., are WFF.
 3. If α and β are two WFFs, then $(\neg\alpha), (\alpha \vee \beta), (\alpha \wedge \beta), (\alpha \rightarrow \beta)$, and $(\alpha \leftrightarrow \beta)$ are WFFs.
 4. If α is a WFF where all occurrences of x are free, then $\forall x\alpha$ and $\exists x\alpha$ are WFFs.

Well-formed Formulas

- Definition*:**
1. Propositional variables such as p, q, r , etc., are WFFs.
 2. Propositional functions such as $P(x), Q(x, y), R(x, y, z)$, etc., are WFF.
 3. If α and β are two WFFs, then $(\neg\alpha), (\alpha \vee \beta), (\alpha \wedge \beta), (\alpha \rightarrow \beta)$, and $(\alpha \leftrightarrow \beta)$ are WFFs.
 4. If α is a WFF where all occurrences of x are free, then $\forall x\alpha$ and $\exists x\alpha$ are WFFs.

Example:

Well-formed Formulas

- Definition*:**
1. Propositional variables such as p, q, r , etc., are WFFs.
 2. Propositional functions such as $P(x), Q(x, y), R(x, y, z)$, etc., are WFF.
 3. If α and β are two WFFs, then $(\neg\alpha), (\alpha \vee \beta), (\alpha \wedge \beta), (\alpha \rightarrow \beta)$, and $(\alpha \leftrightarrow \beta)$ are WFFs.
 4. If α is a WFF where all occurrences of x are free, then $\forall x\alpha$ and $\exists x\alpha$ are WFFs.

Example: $\forall x(p \vee (\neg Q(x)))$ is a WFF.

Well-formed Formulas

- Definition*:**
1. Propositional variables such as p, q, r , etc., are WFFs.
 2. Propositional functions such as $P(x), Q(x, y), R(x, y, z)$, etc., are WFF.
 3. If α and β are two WFFs, then $(\neg\alpha), (\alpha \vee \beta), (\alpha \wedge \beta), (\alpha \rightarrow \beta)$, and $(\alpha \leftrightarrow \beta)$ are WFFs.
 4. If α is a WFF where all occurrences of x are free, then $\forall x\alpha$ and $\exists x\alpha$ are WFFs.

Example: $\forall x(p \vee (\neg Q(x)))$ is a WFF.

p

Well-formed Formulas

- Definition*:**
1. Propositional variables such as p, q, r , etc., are WFFs.
 2. Propositional functions such as $P(x), Q(x, y), R(x, y, z)$, etc., are WFF.
 3. If α and β are two WFFs, then $(\neg\alpha), (\alpha \vee \beta), (\alpha \wedge \beta), (\alpha \rightarrow \beta)$, and $(\alpha \leftrightarrow \beta)$ are WFFs.
 4. If α is a WFF where all occurrences of x are free, then $\forall x\alpha$ and $\exists x\alpha$ are WFFs.

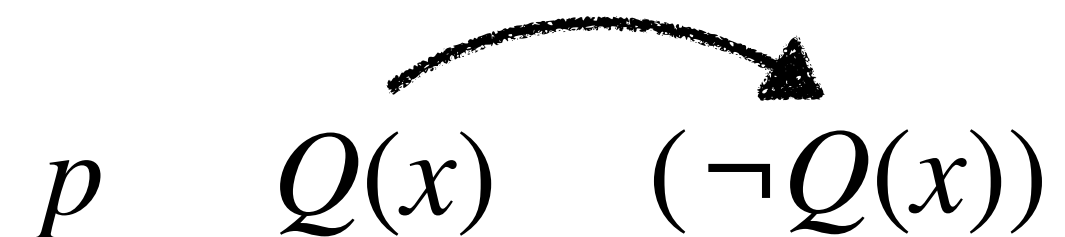
Example: $\forall x(p \vee (\neg Q(x)))$ is a WFF.

$p \quad Q(x)$

Well-formed Formulas

- Definition*:**
1. Propositional variables such as p, q, r , etc., are WFFs.
 2. Propositional functions such as $P(x), Q(x, y), R(x, y, z)$, etc., are WFF.
 3. If α and β are two WFFs, then $(\neg\alpha), (\alpha \vee \beta), (\alpha \wedge \beta), (\alpha \rightarrow \beta)$, and $(\alpha \leftrightarrow \beta)$ are WFFs.
 4. If α is a WFF where all occurrences of x are free, then $\forall x\alpha$ and $\exists x\alpha$ are WFFs.

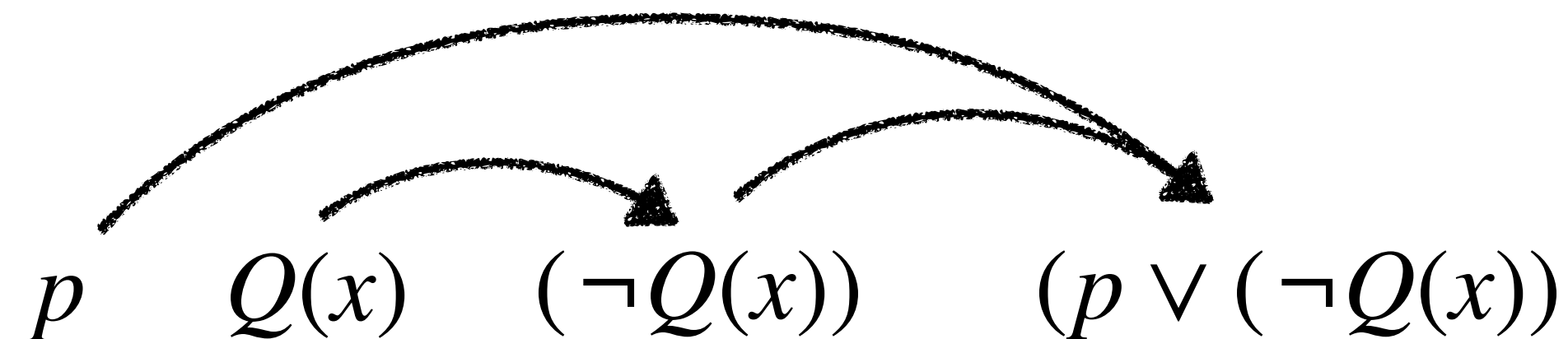
Example: $\forall x(p \vee (\neg Q(x)))$ is a WFF.



Well-formed Formulas

- Definition*:**
1. Propositional variables such as p, q, r , etc., are WFFs.
 2. Propositional functions such as $P(x), Q(x, y), R(x, y, z)$, etc., are WFF.
 3. If α and β are two WFFs, then $(\neg\alpha), (\alpha \vee \beta), (\alpha \wedge \beta), (\alpha \rightarrow \beta)$, and $(\alpha \leftrightarrow \beta)$ are WFFs.
 4. If α is a WFF where all occurrences of x are free, then $\forall x\alpha$ and $\exists x\alpha$ are WFFs.

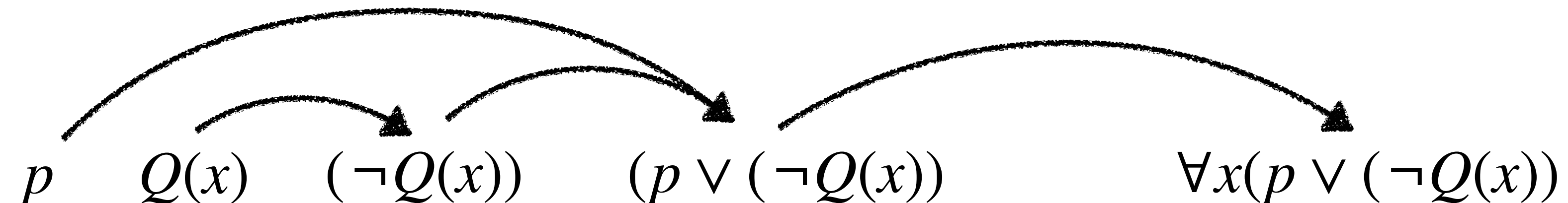
Example: $\forall x(p \vee (\neg Q(x)))$ is a WFF.



Well-formed Formulas

- Definition*:**
1. Propositional variables such as p, q, r , etc., are WFFs.
 2. Propositional functions such as $P(x), Q(x, y), R(x, y, z)$, etc., are WFF.
 3. If α and β are two WFFs, then $(\neg\alpha), (\alpha \vee \beta), (\alpha \wedge \beta), (\alpha \rightarrow \beta)$, and $(\alpha \leftrightarrow \beta)$ are WFFs.
 4. If α is a WFF where all occurrences of x are free, then $\forall x\alpha$ and $\exists x\alpha$ are WFFs.

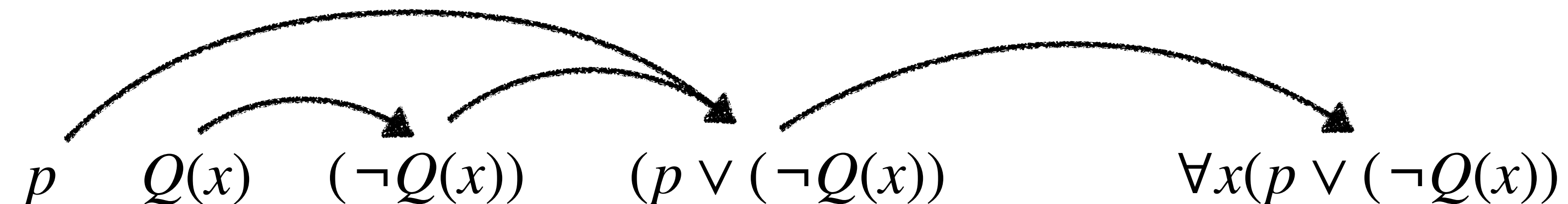
Example: $\forall x(p \vee (\neg Q(x)))$ is a WFF.



Well-formed Formulas

- Definition*:**
1. Propositional variables such as p, q, r , etc., are WFFs.
 2. Propositional functions such as $P(x), Q(x, y), R(x, y, z)$, etc., are WFF.
 3. If α and β are two WFFs, then $(\neg\alpha), (\alpha \vee \beta), (\alpha \wedge \beta), (\alpha \rightarrow \beta)$, and $(\alpha \leftrightarrow \beta)$ are WFFs.
 4. If α is a WFF where all occurrences of x are free, then $\forall x\alpha$ and $\exists x\alpha$ are WFFs.

Example: $\forall x(p \vee (\neg Q(x)))$ is a WFF.

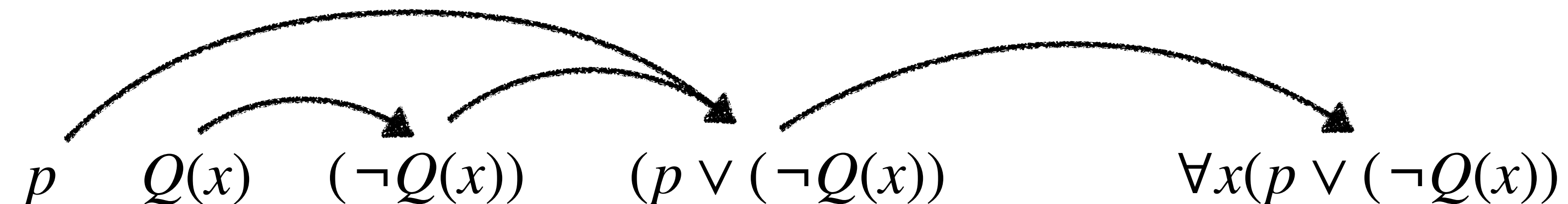


$\forall x(\exists x(P(x)))$ is **not** a WFF.

Well-formed Formulas

- Definition*:**
1. Propositional variables such as p, q, r , etc., are WFFs.
 2. Propositional functions such as $P(x), Q(x, y), R(x, y, z)$, etc., are WFF.
 3. If α and β are two WFFs, then $(\neg\alpha)$, $(\alpha \vee \beta)$, $(\alpha \wedge \beta)$, $(\alpha \rightarrow \beta)$, and $(\alpha \leftrightarrow \beta)$ are WFFs.
 4. If α is a WFF where all occurrences of x are free, then $\forall x\alpha$ and $\exists x\alpha$ are WFFs.

Example: $\forall x(p \vee (\neg Q(x)))$ is a WFF.



$\forall x(\exists x(P(x)))$ is **not** a WFF.

*WFF in FOL are slightly different from how they are defined here.

Precedence of Quantifier

Precedence of Quantifier

\forall and \exists have higher precedence than all the logical operators.

Precedence of Quantifier

\forall and \exists have higher precedence than all the logical operators.

$\forall xP(x) \vee Q(x)$ is $(\forall xP(x)) \vee Q(x)$ not $\forall x(P(x) \vee Q(x))$

Precedence of Quantifier

\forall and \exists have higher precedence than all the logical operators.

$\forall xP(x) \vee Q(x)$ is $(\forall xP(x)) \vee Q(x)$ not $\forall x(P(x) \vee Q(x))$

$\forall xP(x) \rightarrow \exists yQ(y)$ is $(\forall xP(x)) \rightarrow (\exists yQ(y))$ not $\forall x(P(x) \rightarrow \exists yQ(y))$

Binding Variables

Binding Variables

Example:

Binding Variables

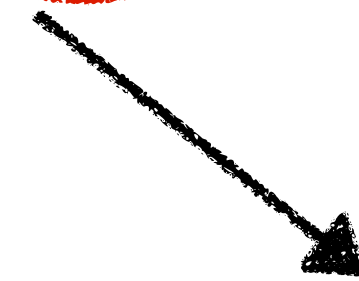
Example:

$$(x + y) = 1$$

Binding Variables

Example:

$$\underline{(x + y) = 1}$$



Not a proposition, both x & y are **free**.

Binding Variables

- An occurrence of a variable that is not bound by a quantifier is called **free**.

Example:

$$\underline{(x + y) = 1}$$

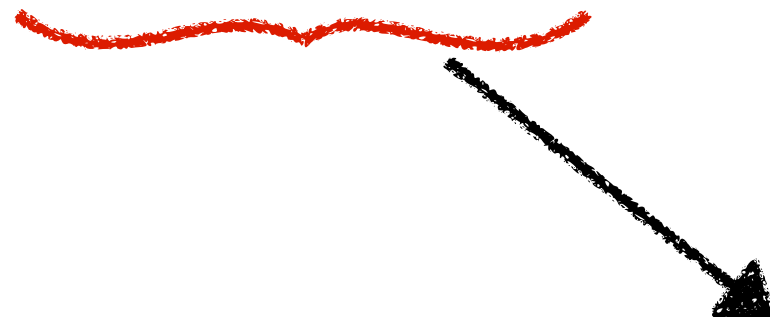


Not a proposition, both x & y are **free**.

Binding Variables

- An occurrence of a variable that is not bound by a quantifier is called **free**.

Example:

$$\exists x (x + y) = 1$$


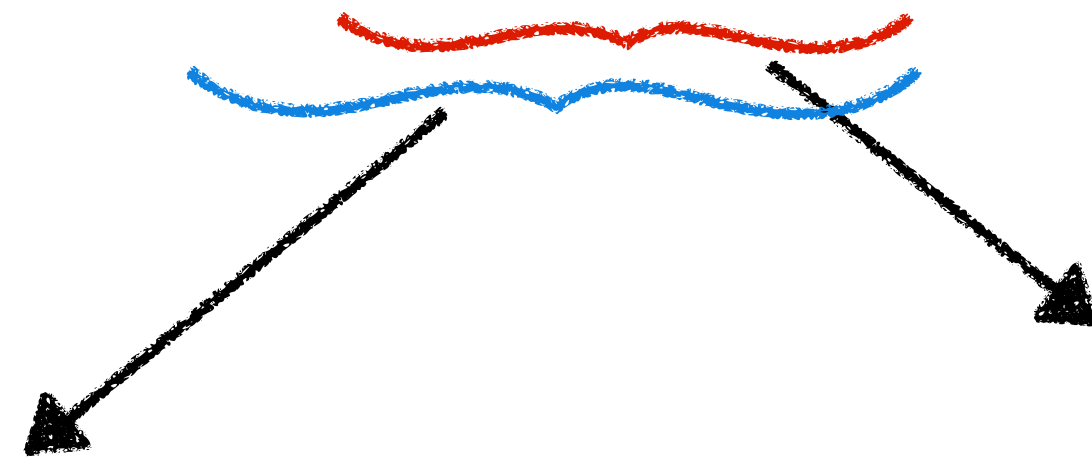
Not a proposition, both x & y are **free**.

Binding Variables

- An occurrence of a variable that is not bound by a quantifier is called **free**.

Example:

$$\exists x (x + y) = 1$$



Not a proposition, x is **bound** but y is **free**.

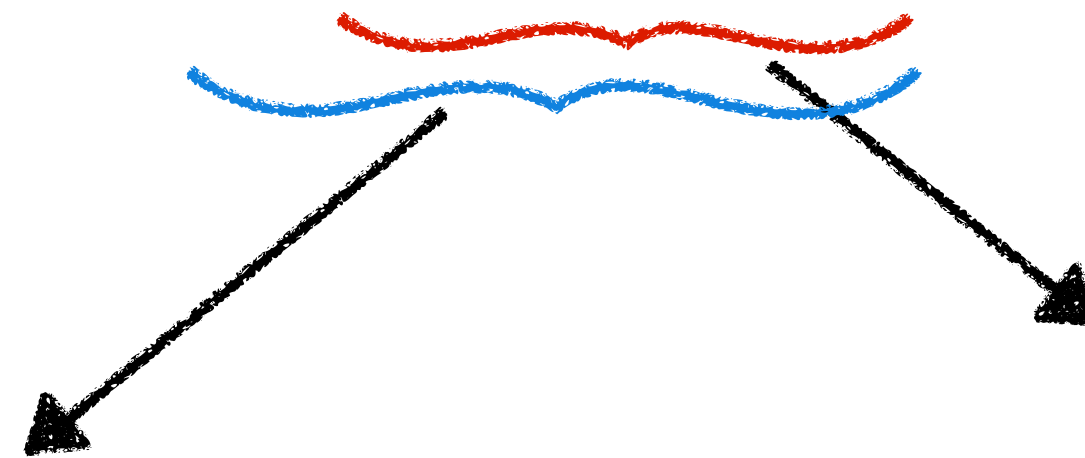
Not a proposition, both x & y are **free**.

Binding Variables

- An occurrence of a variable that is not bound by a quantifier is called **free**.
- An occurrence of a variable that is bound by a quantifier is called **bound**.

Example:

$$\exists x (x + y) = 1$$



Not a proposition, x is **bound** but y is **free**.

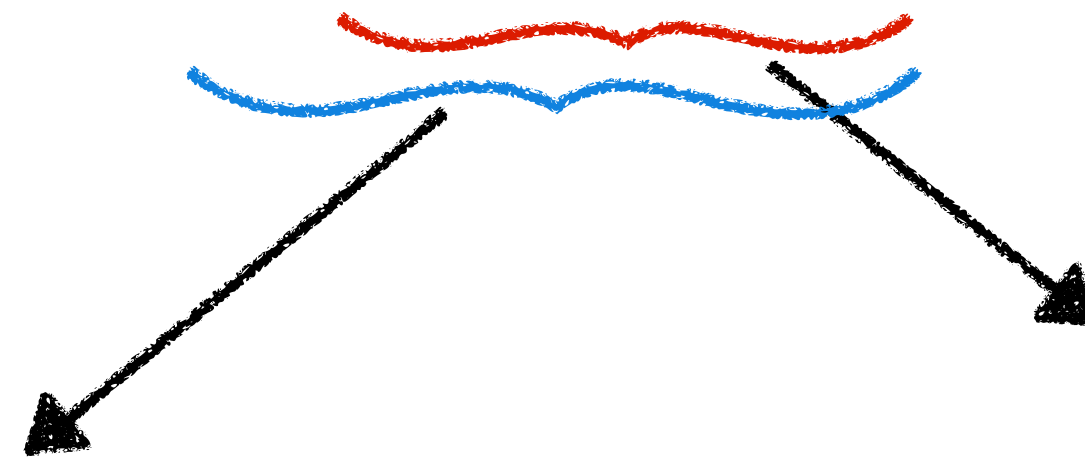
Not a proposition, both x & y are **free**.

Binding Variables

- An occurrence of a variable that is not bound by a quantifier is called **free**.
- An occurrence of a variable that is bound by a quantifier is called **bound**.

Example:

$$\forall y \exists x (x + y) = 1$$



Not a proposition, x is **bound** but y is **free**.

Not a proposition, both x & y are **free**.

Binding Variables

- An occurrence of a variable that is not bound by a quantifier is called **free**.
- An occurrence of a variable that is bound by a quantifier is called **bound**.

Example:

$$\forall y \exists x (x + y) = 1$$



Not a proposition, x is **bound** but y is **free**.

Not a proposition, both x & y are **free**.

A proposition as no variable is **free**.

Scope

Scope

Example:

Scope

Example:

$$\exists x (P(x) \wedge Q(x)) \vee \forall x R(x)$$

Scope

Example:

$$\exists x (P(x) \wedge Q(x)) \vee \forall x R(x)$$



Scope of $\exists x$ is $P(x) \wedge Q(x)$

Scope

Example:

$$\exists x (P(x) \wedge Q(x)) \vee \forall x R(x)$$

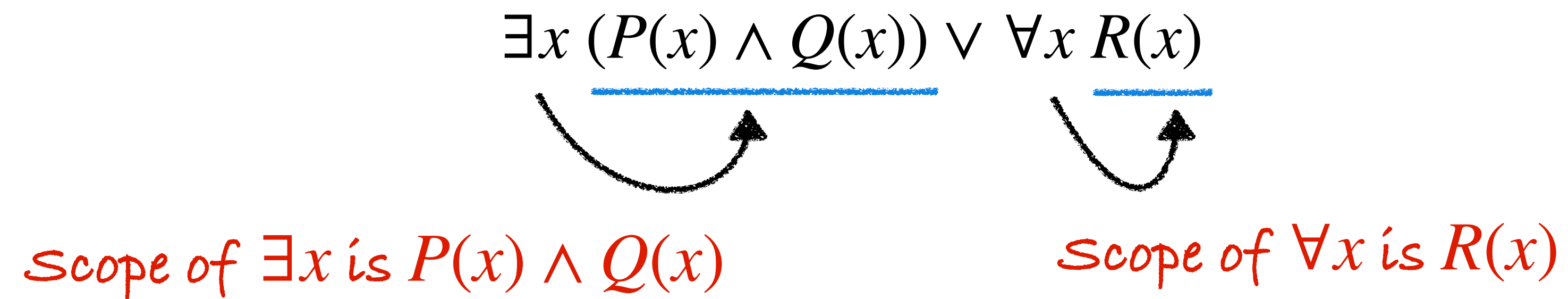
Scope of $\exists x$ is $P(x) \wedge Q(x)$

Scope of $\forall x$ is $R(x)$

Scope

The part of a logical expression where a quantifier is applied is called the **scope** of the quantifier.

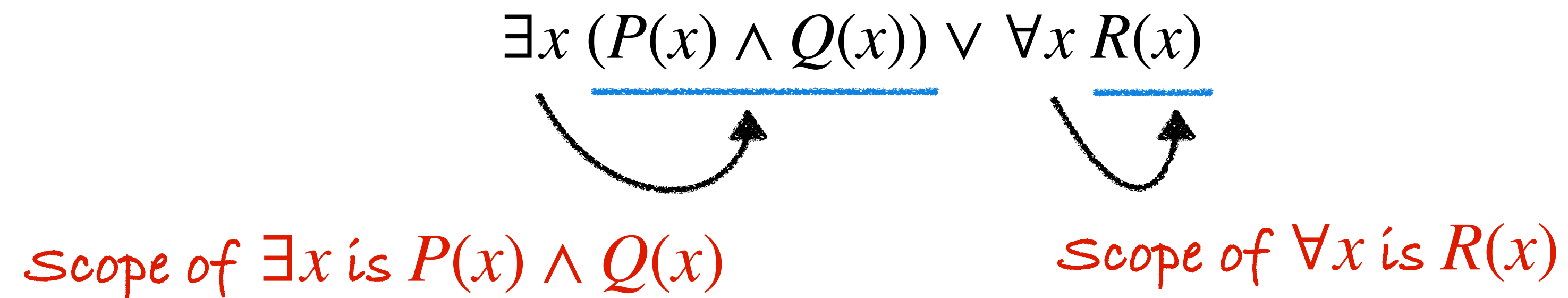
Example:



Scope

The part of a logical expression where a quantifier is applied is called the **scope** of the quantifier.

Example:

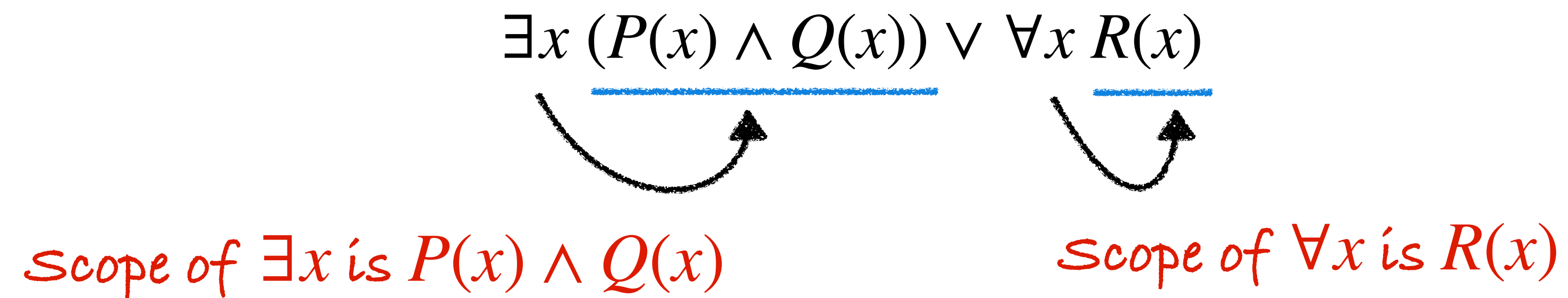


Tip: To avoid confusion, use different variables:

Scope

The part of a logical expression where a quantifier is applied is called the **scope** of the quantifier.

Example:



Tip: To avoid confusion, use different variables:

$$\exists x (P(x) \wedge Q(x)) \vee \forall y R(y)$$

Logical Equivalence of WFFs

Logical Equivalence of WFFs

Definition:

Logical Equivalence of WFFs

Definition: Two WFFs containing same set of propositional variables and propositional

Logical Equivalence of WFFs

Definition: Two WFFs containing same set of propositional variables and propositional functions

Logical Equivalence of WFFs

Definition: Two WFFs containing same set of propositional variables and propositional functions with no free variables are **logically equivalent** if and only if

Logical Equivalence of WFFs

Definition: Two WFFs containing same set of propositional variables and propositional functions with no free variables are **logically equivalent** if and only if they have the same

Logical Equivalence of WFFs

Definition: Two WFFs containing same set of propositional variables and propositional functions with no free variables are **logically equivalent** if and only if they have the same truth value,

Logical Equivalence of WFFs

Definition: Two WFFs containing same set of propositional variables and propositional functions with no free variables are **logically equivalent** if and only if they have the same truth value, no matter which **propositions** and **predicates** are substituted in WFFs and

Logical Equivalence of WFFs

Definition: Two WFFs containing same set of propositional variables and propositional functions with no free variables are **logically equivalent** if and only if they have the same truth value, no matter which **propositions** and **predicates** are substituted in WFFs and which **domain** is used for variables.

Logical Equivalence of WFFs

Definition: Two WFFs containing same set of propositional variables and propositional functions with no free variables are **logically equivalent** if and only if they have the same truth value, no matter which **propositions** and **predicates** are substituted in WFFs and which **domain** is used for variables.

Example:

Logical Equivalence of WFFs

Definition: Two WFFs containing same set of propositional variables and propositional functions with no free variables are **logically equivalent** if and only if they have the same truth value, no matter which **propositions** and **predicates** are substituted in WFFs and which **domain** is used for variables.

Example: $\forall x(P(x) \wedge Q(x))$ is logically equivalent to $\forall xP(x) \wedge \forall xQ(x)$

Logical Equivalence of WFFs

Definition: Two WFFs containing same set of propositional variables and propositional functions with no free variables are **logically equivalent** if and only if they have the same truth value, no matter which **propositions** and **predicates** are substituted in WFFs and which **domain** is used for variables.

Example: $\forall x(P(x) \wedge Q(x))$ is logically equivalent to $\forall xP(x) \wedge \forall xQ(x)$

(Check proof in the book if you are interested.)

Negating Quantified Expressions

Negating Quantified Expressions

q = "Every student in this class has taken ICS."

Negating Quantified Expressions

q = "Every student in this class has taken ICS."

$q = \forall xP(x),$

Negating Quantified Expressions

q = "Every student in this class has taken ICS."

$q = \forall xP(x)$, where $P(x) = "x \text{ has taken ICS}"$

Negating Quantified Expressions

q = "Every student in this class has taken ICS."

$q = \forall x P(x)$, where $P(x)$ = " x has taken ICS" and domain is the set of students in this class.

Negating Quantified Expressions

q = "Every student in this class has taken ICS."

$q = \forall x P(x)$, where $P(x)$ = " x has taken ICS" and domain is the set of students in this class.

What's the negation of q ?

Negating Quantified Expressions

q = "Every student in this class has taken ICS."

$q = \forall x P(x)$, where $P(x)$ = " x has taken ICS" and domain is the set of students in this class.

What's the negation of q ?

$\neg q$ = "It is not the case that every student in this class has taken ICS."

Negating Quantified Expressions

q = "Every student in this class has taken ICS."

$q = \forall x P(x)$, where $P(x)$ = " x has taken ICS" and domain is the set of students in this class.

What's the negation of q ?

$\neg q$ = "It is not the case that every student in this class has taken ICS."

= "There is a student in this class who has not taken ICS."

Negating Quantified Expressions

q = "Every student in this class has taken ICS."

$q = \forall x P(x)$, where $P(x)$ = " x has taken ICS" and domain is the set of students in this class.

What's the negation of q ?

$\neg q$ = "It is not the case that every student in this class has taken ICS."

= "There is a student in this class who has not taken ICS."

= $\exists x \neg P(x)$

Negating Quantified Expressions

q = "Every student in this class has taken ICS."

$q = \forall x P(x)$, where $P(x)$ = " x has taken ICS" and domain is the set of students in this class.

What's the negation of q ?

$\neg q$ = "It is not the case that every student in this class has taken ICS."

= "There is a student in this class who has not taken ICS."

= $\exists x \neg P(x)$

This illustrates the logical equivalence:

Negating Quantified Expressions

q = "Every student in this class has taken ICS."

$q = \forall x P(x)$, where $P(x)$ = " x has taken ICS" and domain is the set of students in this class.

What's the negation of q ?

$\neg q$ = "It is not the case that every student in this class has taken ICS."

= "There is a student in this class who has not taken ICS."

= $\exists x \neg P(x)$

This illustrates the logical equivalence:

$$\neg \forall x P(x) \equiv \exists x \neg P(x)$$

Negating Quantified Expressions

Negating Quantified Expressions

q = "There's a student in M4C who had 100% attendance in ICS."

Negating Quantified Expressions

q = "There's a student in M4C who had 100% attendance in ICS."

$q = \exists x P(x),$

Negating Quantified Expressions

q = "There's a student in M4C who had 100% attendance in ICS."

$q = \exists x P(x)$, where $P(x)$ = " x had 100% attendance in ICS" and domain is set of students in M4C.

Negating Quantified Expressions

q = "There's a student in M4C who had 100% attendance in ICS."

$q = \exists xP(x)$, where $P(x)$ = " x had 100% attendance in ICS" and domain is set of students in M4C.

What's the negation of q ?

Negating Quantified Expressions

q = "There's a student in M4C who had 100% attendance in ICS."

$q = \exists x P(x)$, where $P(x)$ = " x had 100% attendance in ICS" and domain is set of students in M4C.

What's the negation of q ?

$\neg q$ = "It is not the case that there's a student in M4C who had 100% attendance in ICS."

Negating Quantified Expressions

q = "There's a student in M4C who had 100% attendance in ICS."

$q = \exists x P(x)$, where $P(x)$ = " x had 100% attendance in ICS" and domain is set of students in M4C.

What's the negation of q ?

$\neg q$ = "It is not the case that there's a student in M4C who had 100% attendance in ICS."

= "Every student in M4C did not have 100% attendance in ICS."

Negating Quantified Expressions

q = "There's a student in M4C who had 100% attendance in ICS."

$q = \exists x P(x)$, where $P(x)$ = " x had 100% attendance in ICS" and domain is set of students in M4C.

What's the negation of q ?

$\neg q$ = "It is not the case that there's a student in M4C who had 100% attendance in ICS."

= "Every student in M4C did not have 100% attendance in ICS."

= $\forall x \neg P(x)$

Negating Quantified Expressions

q = "There's a student in M4C who had 100% attendance in ICS."

$q = \exists x P(x)$, where $P(x)$ = " x had 100% attendance in ICS" and domain is set of students in M4C.

What's the negation of q ?

$\neg q$ = "It is not the case that there's a student in M4C who had 100% attendance in ICS."

= "Every student in M4C did not have 100% attendance in ICS."

= $\forall x \neg P(x)$

This illustrates the logical equivalence:

$$\neg \exists x P(x) \equiv \forall x \neg P(x)$$